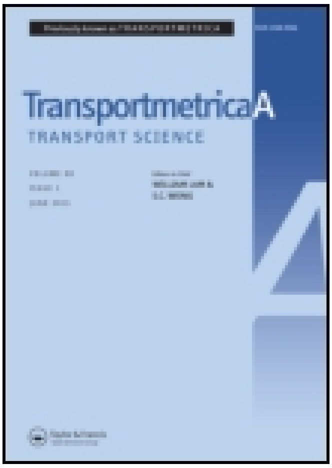

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Methodological issues in modelling time-of-travel preferences

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We address three methodological issues that arise when modelling time-of-travel preferences: unequal period lengths, schedule delay in the absence of desired time-of-travel data and the 24-hour cycle. Varying period length is addressed by using size variables. Schedule delay is treated by assuming either arrival or departure time sensitivity and using market segment specific utility functions of time-of-travel, or using distributions of the desired times-of-travel. The 24-hour cycle is modelled by using a trigonometric utility functional form. These methodologies are demonstrated in the context of a tour-based travel demand model using the 2000 Bay Area travel survey.

Keywords: time-of-travel modelling; time of day modelling; schedule delay; cyclicality; tour-based model

1. Introduction

1.1. *Motivation and objective*

Modelling time-of-travel preferences is required for the prediction of transportation system performance and the evaluation of policies, such as congestion pricing. We advance the state of the art of modelling time-of-travel preferences by addressing three methodological issues: (1) modelling time periods of unequal length, (2) accounting for schedule delay when data on the desired times-of-travel are unavailable and (3) modelling the cyclical properties of time-of-travel preferences.

The first issue arises because of the discretisation of continuous time into time intervals or periods. These time intervals are often of varying length, for example, due to the small volume of trips observed during certain time intervals. We show how to specify a time-of-travel model that accounts for time periods of unequal length.

The second issue is related to schedule delay, which is a fundamental concept in modelling the time-of-travel choice (Kraft and Wohl 1967, Vickrey 1969, Cosslett 1977, Abkowitz 1980, Hendrickson and Kocur 1981, Small 1982). It postulates that travellers

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have desired arrival or departure times, and that travel at other times incurs disutility. If the desired times-of-travel were known, it would be relatively straightforward to include schedule delay in a time-of-travel choice model. However, these data are often not collected especially in revealed preferences surveys and are difficult to forecast, and therefore it is important to find methods that account for schedule delay when it is not observed.

The third issue arises because of the 24-hour cycle and its implications on time-of-travel preferences. That is, for a model designed for a one-day time frame, times t and $t + 24$ hours exhibit the same time-of-travel preference and should have the same utility of arrival (or departure). We wish to address how to account for these cyclical properties in the specification of the time-of-travel model.

1.2. *Literature review*

Time-of-travel choice has been studied using different methodological approaches which vary by the level of temporal analysis, model structure and type of data collected. These various approaches include the use of discrete (see, e.g. Small 1982) versus continuous time models (Wang 1996, van Vuren *et al.* 1999); model structures ranging from logit to other more general models such as nested logit (Brownstone and Small 1989), ordered generalised extreme value (Small 1987), bivariate and multinomial probit (Liu and Mahmassani 1998, Lemp *et al.* 2011), error components logit (Bhat 1998, de Jong *et al.* 2003, RAND Europe 2005, Hess *et al.* 2007, Holyoak 2007, Kristoffersson and Engelson 2008), continuous hazard-based logit (Bhat and Steed 2002), continuous cross-nested logit (Lemp *et al.* 2010), and the presence of schedule delay terms, generally available from stated preferences but not revealed preferences surveys (see, e.g. de Jong *et al.* 2003).

Next we discuss how the literature has generally treated the methodological issues raised in this article. First, the number and length of the time periods used have varied, with earlier efforts using a small number of coarse time periods and more recent work using a larger number of time periods. For example, in RAND Europe (2005), Hess *et al.* (2007) and Kristoffersson and Engelson (2008), periods as short as 1 h or 15 min are used in the model. Generally, time period-specific constants for arrival time, departure time and/or duration are included in the utility equations of the time periods. If the periods are of unequal length, these constants will capture the effect of unequal lengths but will mask the pattern of time-of-travel preferences, and therefore the use of size variables is preferred, as discussed later.

Second, schedule delay information has generally been included in time-of-travel models estimated from stated preferences data where information about preferred times-of-travel is likely to be collected. However, when these models are used in application or when models are estimated from revealed preferences data, the schedule delay terms are normally excluded since information about scheduling preferences are unavailable or difficult to forecast (see, e.g. Hess *et al.* 2007). The inherent assumption is that the alternative-specific constants will capture these schedule delay effects, among other things. However, as we show later in this article, the constants capture the effects of schedule delay only if additional assumptions are employed. Using revealed preferences data, Koppelman *et al.* (2008) include a variable defined as the weighted average of the differences between a departure time alternative and all time periods, where the weights capture departure time preferences for different periods. Kristoffersson and Engelson (2008) address the problem of applying a previously estimated model with schedule delay in situations where there are no data on preferred times-of-travel. They present an approach to use the model and observed times-of-travel to estimate aggregate distributions of preferred times-of-travel.

Third, functional forms have been developed to approximate alternative-specific constants so as to avoid overfitting and identification problems as the number of these constants increases with the number of time periods. For example, in Hess *et al*. (2005), exponential, power and empirical functions are used to approximate these constants. However, to the best of our knowledge, the cyclical properties of time-of-travel preferences have not been dealt with in developing time-of-travel choice models except in recent work referenced in the following section that has built on the methodology described in this article. In addition, Grammig *et al*. (2005) have explored a similar idea to the one we have proposed.

1.3. *Contributions and organisation*

The contributions of this research are (1) the use of size variables to account for unequal period lengths, (2) the development of methods which obviate the need for explicitly incorporating schedule delay in the utilities of the time period alternatives while accounting for its effect on the utility, (3) the use of continuous cyclic functions of time which ensure that the utility at a time t is equal to the utility at time $t + 24$ hours and (4) the demonstration of the developed methods empirically using a tour-based travel demand modelling approach for the San Francisco Bay Area. These methodological issues and their solutions have been developed by the authors of this article for a project whose results are documented in Cambridge Systematics, Inc. (2005) and Abou Zeid *et al*. (2006). The project aimed to develop practical time-of-travel forecasting methods that can be used to supplement existing operational travel demand model systems developed using standard transportation planning surveys. The purpose of this article is to provide the detailed derivations and analyses.

The methodologies developed in this article have been used and sometimes extended in several applications. A case study in Tel-Aviv uses our modelling approach to model time-of-travel in a tour-based context (Popuri *et al*. 2008). Other papers apply only the cyclicality property (see, e.g. Lemp *et al*. 2010). Carrier (2008) models time-of-travel choice for airline travellers by using trigonometric functions to represent time as a continuous variable. Brey and Walker (2011) apply one of the methods we suggest for dealing with unobserved desired time-of-travel and extend the method, as will be described later in this article.

The remainder of this article is organised as follows. Section 2 discusses the issue of unequal period lengths and proposes a method to account for it. Section 3 develops methods for incorporating schedule delay when desired times-of-travel are unobserved. Section 4 derives the continuous functions of time that satisfy the cyclicality property of time-of-travel. Section 5 presents a case study describing the application of these methodological issues to the San Francisco Bay Area and presents selected model estimation results to illustrate the concepts. Section 6 concludes this article.

2. Modelling time periods of unequal length

In discrete choice models of time-of-travel, a number of alternatives are defined which could be of varying lengths. In this section, we discuss how to account for this issue in the utility specification of the model.

Let t be an index for continuous time, where $t \in [0, 24]$. Let $v(t)$ denote the systematic utility of time-of-travel t and $f(t)$ be the probability density function of t. For a continuous logit model (Ben-Akiva and Watanatada 1981, de Palma *et al.* 1983), $f(t)$ is given by

$$f(t) = \frac{e^{v(t)}}{\int_0^{24} e^{v(t')} \mathrm{d}t'} \tag{1}$$

We discretise the 24-hour time horizon into H periods. For a time period h where $h = 1, \ldots, H$, let $t_s(h)$ denote its start time (with respect to an arbitrary reference point), Δ_h its length and $P(h)$ the choice probability of time period h. $P(h)$ can be expressed as follows:

$$P(h) = \int_{t_s(h)}^{t_s(h)+\Delta_h} \frac{e^{v(t)}}{\int_0^{24} e^{v(t')} \mathrm{d}t'} \mathrm{d}t = \frac{\int_{t_s(h)}^{t_s(h)+\Delta_h} e^{v(t)} \mathrm{d}t}{\int_0^{24} e^{v(t')} \mathrm{d}t'} \tag{2}$$

Applying the mean-value theorem for integrals, define for the interval $[t_s(h), t_s(h) + \Delta_h]$ the systematic utility $V(h)$ of period h, equal to the value of $v(t)$ at a 'mid-point'[1] of time interval h, and express (2) as follows:

$$P(h) = \frac{e^{V(h)}\Delta_h}{\int_0^{24} e^{v(t')} \mathrm{d}t'} = \frac{e^{V(h)}\Delta_h}{\sum_{h'=1}^{H} e^{V(h')}\Delta_{h'}} = \frac{e^{V(h)+\ln \Delta_h}}{\sum_{h'=1}^{H} e^{V(h')+\ln \Delta_{h'}}} \tag{3}$$

Thus, time periods of unequal length can be accounted for by adding the natural logarithm of the length of the period (size variable) to its systematic utility and constraining the coefficient of the size variable to 1.

3. Accounting for schedule delay

Schedule delay is a fundamental concept in modelling time-of-travel choice which captures the disutility caused by travelling at times other than the desired times-of-travel. In this section, we discuss two approaches that can be used to account for schedule delay when data on the desired times-of-travel are unavailable.

Let h denote a time-of-travel period, h^* denote a desired time-of-travel period, a denote an arrival time period, a^* denote a desired arrival time period, d denote a departure time period, d^* denote a desired departure time period, $TT(h)$ denote the travel time in period h and $SD(h, h^*)$ denote the schedule delay for travel period h given a desired time-of-travel period h^*. Let t denote a time-of-travel, t^* denote a desired time-of-travel, $tt(t)$ denote the travel time corresponding to time-of-travel t and $sd(t, t^*)$ denote the schedule delay for time-of-travel t given a desired time-of-travel t^*.

3.1. *Approach 1: assume constant desired times-of-travel by market segment*

It can be assumed that some trips are associated with a desired arrival time, such as the trip from home to work, while other trips are associated with a desired departure time such as the trip from work to home. The idea that we pursue is to try to reduce schedule delay to a constant. We wish to prove that, for trips that are arrival time sensitive, modelling arrival time choice reduces schedule delay to a constant if desired arrival time is assumed to be constant for individuals in a market segment. Similarly, for trips that are departure time sensitive, modelling departure time choice reduces schedule delay to a constant if the desired departure time is assumed to be constant for individuals in a market segment.

3.1.1. *Arrival time sensitive trips*

Consider first trips that are arrival time sensitive, i.e. where the individual has a desired arrival time period a^*. We specify the systematic utility of a time-of-travel period as a function of the travel time, schedule delay and size of the period. We also include an alternative-specific constant and allow the specification to include other explanatory variables. The systematic utility of an arrival time period a can be expressed as follows:

$$V(a) = \alpha_1(a) + \beta_1 TT(a) + \gamma_1 SD(a, a^*) + \ln \Delta_a + \cdots \quad (4)$$

where $\alpha_1(a)$ is an alternative-specific constant, β_1 and γ_1 are coefficients to be estimated and $\ln \Delta_a$ is the size variable described in the previous section.

For a desired arrival time period a^*, modelling arrival time choice means that $\gamma_1 SD(a, a^*)$, which is a function of the difference between a and a^*, can be expressed as a function $g_1(a)$ for a given market segment if a^* is assumed to be constant for individuals in that market segment. $g_1(a)$ is then an attribute of period a (whose value does not vary across individuals in a market segment) and is absorbed by the alternative-specific constant (for the respective market segment) of the systematic utility of period a; in this case, there is no need to explicitly include a schedule delay term in the systematic utility as long as an alternative-specific constant for the respective market segment is included.

Suppose, on the other hand, that departure time choice is modelled for a trip with desired arrival time. The utility of departing at time t_d will include a schedule delay term $sd(t_a, t_{a^*})$, where t_a is the arrival time which corresponds to a departure time t_d and is given by $t_a = t_d + tt(t_d)$. In this case, the schedule delay, which is a function of the difference between t_a and t_{a^*}, would depend on t_d, travel time $tt(t_d)$, and t_{a^*}. Even if t_{a^*} is assumed to be constant for individuals in a market segment, the travel time $tt(t_d)$ will assume a different value for individuals in that market segment who travel between different origins and destinations. Therefore, schedule delay would depend on travel time and cannot be a constant.

3.1.2. *Departure time sensitive trips*

For trips with a desired departure time, the systematic utility of a departure time period d can be expressed as follows:

$$V(d) = \alpha_2(d) + \beta_2 TT(d) + \gamma_2 SD(d, d^*) + \ln \Delta_d + \cdots \quad (5)$$

where $\alpha_2(d)$ is an alternative-specific constant, β_2 and γ_2 are coefficients to be estimated, and $\ln \Delta_d$ is a size variable. By a similar argument as above, for trips with a desired departure time, modelling departure time choice reduces the schedule delay $SD(d, d^*)$ to a constant if the desired departure time d^* is assumed to be constant for individuals in a market segment.

To sum up, we model arrival time choice if the trip is arrival sensitive (i.e. with a desired arrival time) and model departure time choice if the trip is departure sensitive (i.e. with a desired departure time). Schedule delay functions become arrival- and departure-specific constants by market segment.

This approach assumes that it is known whether a trip has a desired arrival time or a desired departure time. It may be that for a given trip (e.g. from home to work) some individuals have a desired arrival time while others have a desired departure time. Moreover, the approach employs *a priori* segmentation of constant desired times-of-travel based on socio-economic or demographic characteristics. These two assumptions can be relaxed using latent segmentation, e.g. through latent classes and Hybrid Choice models (Ben-Akiva *et al*. 2002, Walker and Ben-Akiva 2002). We do not attempt to do so here as the objective of this article is to develop practical methods that can be easily integrated within operational travel demand model systems and using standard travel surveys. However, the second approach discussed next briefly describes how a latent desired time-of-travel can be modelled.

3.2. *Approach 2: latent desired times-of-travel*

An alternative approach to the one described above is to assume a probability density function $f(t^*)$ for the latent (unobserved) desired time-of-travel t^* (arrival or departure time, as appropriate) such that:

$$\int_0^{24} f(t^*)\mathrm{d}t^* = 1 \tag{6}$$

and

$$f(0) = f(24) \tag{7}$$

where Equation (7) assumes a cycle length for time-of-travel preferences of 24 h.

Let $P(h|t^*)$ denote a time-of-travel choice model with an explicit schedule delay term that depends on t^*. Then, the time-of-travel choice probability $P(h)$ can be computed by integrating the conditional choice probability $P(h|t^*)$ over the density of the desired time-of-travel as follows:

$$P(h) = \int_0^{24} P(h|t^*)f(t^*)\mathrm{d}t^* \tag{8}$$

Brey and Walker (2011) apply this method in the context of airline itinerary choice using a stated preferences survey and extend it by using a Hybrid Choice model framework, where the desired time-of-travel is explained by trip and individual characteristics and is measured by indicators from the survey (stated desired time-of-travel).

4. Modelling the 24-hour cycle

In this section, we discuss the specification of the alternative specific constants of the model. These constants are specified as continuous functions of time to smooth the discontinuities in the utility function that would result if dummy variables for the periods were used instead and to reduce the number of unknown parameters which need to be estimated, especially if the data do not contain observations for all arrival and departure time periods.

The contribution of our specification is accounting for the cyclicality of time-of-travel. The cycle length for weekday urban trips is 24 h. The implication of this observation is that the utility of arrival (departure) at a time t should be equal to the utility of arrival (departure) at time $t + 24$ hours. Therefore, in addition to using continuous functions of time, as discussed above, these functions need to satisfy the cyclicality property. We discuss below one type of function which can be used for this purpose. The proposed approach can be used to model cycles of lengths other than 24 h. For example, in the context of time-of-travel choice for airline itineraries, Carrier (2008) models the cyclicality of time-of-travel preferences and additionally estimates the cycle lengths which are found to be 16 h for overnight bookings and 9 h for day trip bookings.

4.1. *Trigonometric function*

We make use of the property that for any trigonometric function $y(\cdot)$, we have $y(0) = y(2k\pi)$, where $k \in Z^+$. Since for our application we require that $v(0) = v(24)$, we define a mapping function $z_k(t)$ that maps $t = 0$ to 0 and $t = 24$ to $2k\pi$ as follows:

$$z_k(t) = \frac{2k\pi t}{24}, \quad 0 \leq t \leq 24, k \in Z^+ \tag{9}$$

with $z_k(0) = 0$ and $z_k(24) = 2k\pi$.

Therefore, a utility function which is a trigonometric function of the mapped arguments will then guarantee that $v(0) = v(24)$. Consider, for example, the following function, which is based on the idea of the Fourier series (Fourier 1822):

$$\begin{aligned} v(t) = \alpha_1 \sin\left(\frac{2\pi t}{24}\right) + \alpha_2 \sin\left(\frac{4\pi t}{24}\right) + \cdots + \alpha_K \sin\left(\frac{2K\pi t}{24}\right) \\ + \alpha_{K+1} \cos\left(\frac{2\pi t}{24}\right) + \alpha_{K+2} \cos\left(\frac{4\pi t}{24}\right) + \cdots + \alpha_{2K} \cos\left(\frac{2K\pi t}{24}\right) \end{aligned} \tag{10}$$

For sufficiently large K this series can be used to approximate any cyclical function. The coefficients $\alpha_1, \ldots, \alpha_{2K}$ need to be estimated from data.

Letting t_h denote the mid-point of time period h (measured from some arbitrary reference point), the utility of arrival or departure in period h can be expressed as follows, where the mid-point of a time period is used to represent the period:

$$\begin{aligned} V(h) &= v(t_h) \\ &= \alpha_1 \sin\left(\frac{2\pi t_h}{24}\right) + \alpha_2 \sin\left(\frac{4\pi t_h}{24}\right) + \cdots + \alpha_K \sin\left(\frac{2K\pi t_h}{24}\right) \\ &\quad + \alpha_{K+1} \cos\left(\frac{2\pi t_h}{24}\right) + \alpha_{K+2} \cos\left(\frac{4\pi t_h}{24}\right) + \cdots + \alpha_{2K} \cos\left(\frac{2K\pi t_h}{24}\right) \end{aligned} \tag{11}$$

This utility function satisfies the cyclicality property since $v(t) = v(t + 24)$. Note that this trigonometric function is specified as a combination of sines and cosines of angles with different frequencies. Having both sines and cosines in the formulation (as opposed to having only sines or cosines) is needed to ensure that every time t between 0 and 24 will have a unique utility value. Moreover, the use of angles with different frequencies is needed to get a better model fit compared to using only one frequency. The truncation point K could be determined empirically based on the resulting profile of the utility function and the statistical significance of the terms comprising the function. Including socio-economic variables in the utility can be done by interacting them with a trigonometric function as in the above expression. This is demonstrated in the case study in Section 5.

4.2. *Other functions*

Other functions that guarantee the cyclicality property of time-of-travel may be used. One example is the piecewise linear function with additional constraints on the slopes to ensure that the cyclicality property is preserved (see, e.g. Abou Zeid *et al.* 2006). This function is however not smooth; a piecewise quadratic function with constraints ensures both cyclicality and smoothness.

5. Empirical results

In this section, we describe a case study that shows the application of the above modelling methods to the San Francisco Bay Area using the 2000 travel survey. We first present an overview of the approach used and then show selected model estimation results. Additional details on the methodology, model estimation, and application can be found in Cambridge Systematics (2005) and Abou Zeid *et al.* (2006).

5.1. *Overview of the modelling approach*

We use the 2000 Bay Area travel survey to estimate logit time-of-travel choice models using a tour-based approach. The models are estimated for auto tours/trips of different purposes: work, school, shopping, eat-out, personal business, pick-up/drop-off, discretionary and work-based (subtours). The explanatory variables used include the level of service variables (such as travel time), demographic variables, mode (drive-alone vs. carpool), etc. A total of 35 time periods are used, all of which are half-hours except for the first and last periods (early morning and late evening hours) which are of longer duration.

Time-of-travel choice modelling is done at two levels: primary activity and secondary activity. A primary activity of the tour can be defined to be the activity of longest duration on the tour, the activity with highest priority, etc.; all other activities are considered secondary.

The primary activity divides the tour into two half-tours. Since scheduling decisions on a tour are interrelated, the two half-tours comprising a tour are scheduled simultaneously. We assume that the half-tour from home to the primary activity is arrival time sensitive, while the half-tour from the primary activity to home is departure time sensitive. Therefore, we model the joint choice of arrival time and departure time at the primary activity. Since there are 35 time periods, scheduling the tour at this level involves a choice among 630 alternatives (equal to $35\times(35+1)/2$). An alternative (a, d) is thus characterised by an arrival time period a, a departure time period d and a duration $t_d - t_a$, and its systematic utility can be expressed as follows:

$$\begin{aligned} V(a,d) = {} & \alpha_1(a) + \beta_1 TT(a) + \ln \Delta_a \\ & + \alpha_2(d) + \beta_2 TT(d) + \ln \Delta_d \\ & + \alpha_3(t_d - t_a) + \cdots \end{aligned} \quad (12)$$

where we have included size variables for the arrival and departure time periods, defined as the number of half-hour periods within a given time period, since not all periods are of equal duration. The schedule delay terms for both half-tours are accounted for by alternative-specific constants by market segment, assuming that desired times-of-travel are constant by market segment.

For secondary activities before the primary activity, we can compute (in model application) the departure time t_d^s from the secondary activity given the modelled arrival time t_a (corresponding to period a) at the primary activity, as follows: $t_d^s + tt(t_d^s) = t_a$, where $tt(t_d^s)$ is the travel time corresponding to departure time t_d^s. Time-of-travel choice for secondary activities before the primary activity is then a choice of arrival time from a choice set of at most 35 time periods (periods corresponding to arrival times larger than t_d^s will be unavailable).

Similarly, for secondary activities after the primary activity, we can compute (in model application) the arrival time t_a^s at the secondary activity given the modelled departure time t_d (corresponding to period d) at the primary activity and the travel time $tt(t_d)$ corresponding to departure time t_d, as follows: $t_a^s = t_d + tt(t_d)$. Time-of-travel choice for secondary activities after the primary activity is then a choice of departure time from a choice set of at most 35 time periods (periods corresponding to departure times smaller than t_a^s will be unavailable).

5.2. *Selected model estimation results*

The original 2000 San Francisco Bay Area travel dataset included survey data of 36,680 individuals from 15,064 households. The presentation in this section focuses on time-of-travel preferences related to home-based work tours, and after cleaning the data, the number of observations used in this model is 11,405 tours.

The utility of an (arrival time period a, departure time period d) combination for the home-based work tour is specified as the sum of an arrival time component, a departure time component and a duration component as follows:

$$V(a,d) = V^a(t_a) + V^d(t_d) + V^{dur}(t_d - t_a) \quad (13)$$

where t_a is the arrival time corresponding to period a (taken as the mid-point of period a) and t_d is the departure time corresponding to period d (taken as the mid-point of period d); $V^a(t_a)$, $V^d(t_d)$, and $V^{dur}(t_d - t_a)$ are the arrival time, departure time, and duration components of the systematic utility, respectively.

Define the trigonometric functions $s^{ar}(t_a)$ and $s^{dr}(t_d)$ as follows:

$$\begin{aligned} s^{ar}(t_a) = {} & \alpha_1^{ar} \sin\left(\frac{2\pi t_a}{24}\right) + \alpha_2^{ar} \sin\left(\frac{4\pi t_a}{24}\right) + \alpha_3^{ar} \sin\left(\frac{6\pi t_a}{24}\right) + \alpha_4^{ar} \sin\left(\frac{8\pi t_a}{24}\right) \\ & + \alpha_5^{ar} \cos\left(\frac{2\pi t_a}{24}\right) + \alpha_6^{ar} \cos\left(\frac{4\pi t_a}{24}\right) + \alpha_7^{ar} \cos\left(\frac{6\pi t_a}{24}\right) + \alpha_8^{ar} \cos\left(\frac{8\pi t_a}{24}\right) \end{aligned} \quad (14)$$

$$s^{dr}(t_d) = \alpha_1^{dr}\sin\left(\frac{2\pi t_d}{24}\right) + \alpha_2^{dr}\sin\left(\frac{4\pi t_d}{24}\right) + \alpha_3^{dr}\sin\left(\frac{6\pi t_d}{24}\right) + \alpha_4^{dr}\sin\left(\frac{8\pi t_d}{24}\right) + \alpha_5^{dr}\cos\left(\frac{2\pi t_d}{24}\right) + \alpha_6^{dr}\cos\left(\frac{4\pi t_d}{24}\right) + \alpha_7^{dr}\cos\left(\frac{6\pi t_d}{24}\right) + \alpha_8^{dr}\cos\left(\frac{8\pi t_d}{24}\right) \quad (15)$$

$V^a(t_a)$ and $V^d(t_d)$ are then specified as follows:

$$V^a(t_a) = \sum\nolimits_{r=1}^{8} x_r s^{ar}(t_a) + \beta_1 * \text{travel time} + \ln(\text{number of half} - \text{hour periods in period } a) \quad (16)$$

$$V^d(t_d) = \sum\nolimits_{r=1}^{7} x_r s^{dr}(t_d) + \beta_2 * \text{travel time} + \ln(\text{number of half} - \text{hour periods in period } d) \quad (17)$$

where: $x_1 = 1$; $x_2 =$ part-time worker dummy; $x_3 =$ no work flexibility dummy; $x_4 =$ female with kids dummy; $x_5 =$ high household income dummy; $x_6 =$ distance; $x_7 =$ shared-ride dummy; $x_8 =$ bridge crossing dummy. The distance, shared-ride dummy, bridge crossing dummy and travel time variables are defined for the home to work direction in the arrival time utility function (Equation (16)) and for the work to home direction in the departure time utility function (Equation (17), except for the bridge-crossing dummy which is not included in the departure time utility.

The specification of $V^a(t_a)$ and $V^d(t_d)$ thus includes a 'base' alternative-specific constant net of all interactions with other variables (e.g. the terms $\alpha_1^{a1}\sin\left(\frac{2\pi t_a}{24}\right) + \cdots + \alpha_8^{a1}\cos\left(\frac{8\pi t_a}{24}\right)$ in the arrival time utility), alternative-specific constants by market segment (e.g. part-time worker dummy * $\left[\alpha_1^{a2}\sin\left(\frac{2\pi t_a}{24}\right) + \cdots + \alpha_8^{a2}\cos\left(\frac{8\pi t_a}{24}\right)\right]$ in the arrival time utility), travel time and a size variable.

The duration component $V^{dur}(t_d - t_a)$ is specified as a power series expansion as follows:

$$V^{dur}(t_d - t_a) = \alpha_1^{dur}(t_d - t_a) + \alpha_2^{dur}(t_d - t_a)^2 + \alpha_3^{dur}(t_d - t_a)^3 + \alpha_4^{dur}(t_d - t_a)^4 + \alpha_5^{dur}(t_d - t_a)^5 + \alpha_6^{dur}(t_d - t_a)^6 + \alpha_7^{dur}(t_d - t_a)^7 \quad (18)$$

In the above utility equations, the choices of variables to include, the truncation point $K = 4$ in the trigonometric functions of arrival time and departure time and the degree 7 of the power series expansion of the duration function are based on empirical considerations.

The unknown parameters to be estimated are: $\alpha_1^{ar}, \ldots, \alpha_8^{ar}$ and $\alpha_1^{dr}, \ldots, \alpha_8^{dr}$ for every variable x_r that is interacted with $s^{ar}(t_a)$ and $s^{dr}(t_d)$ in the arrival and departure utility functions, the travel time parameters β_1 and β_2 and the parameters $\alpha_1^{dur}, \ldots, \alpha_7^{dur}$ in the duration utility function. The model was estimated using standard logit estimation software. The model statistics are shown in Table 1.

Figures 1 and 2 show the utility function values corresponding to the estimated departure time component of the systematic utility for work tours. The 'base' plot represents the utility component $\alpha_1^{d1}\sin\left(\frac{2\pi t_d}{24}\right) + \cdots + \alpha_8^{d1}\cos\left(\frac{8\pi t_d}{24}\right)$, which is not interacted with other variables. All other plots represent the sum of the 'base' departure time utility and the alternative-specific constant for a certain market segment (e.g. the plot labelled part-time worker represents $\alpha_1^{d1}\sin\left(\frac{2\pi t_d}{24}\right) + \cdots + \alpha_8^{d1}\cos\left(\frac{8\pi t_d}{24}\right) + \alpha_1^{d2}\sin\left(\frac{2\pi t_d}{24}\right) + \cdots + \alpha_8^{d2}\cos\left(\frac{8\pi t_d}{24}\right)$). The values depicted are relative utilities, computed as the utility components described above divided by the utility function value at 8 AM.

Table 1. Statistics for the home-based work tour time-of-travel choice model.

Number of observations	11,405
Log-likelihood with zero coefficients	−73,513.43
Final value of log-likelihood	−56,842.83
Number of estimated parameters	129
Rho-squared	0.227
Adjusted rho-squared	0.225

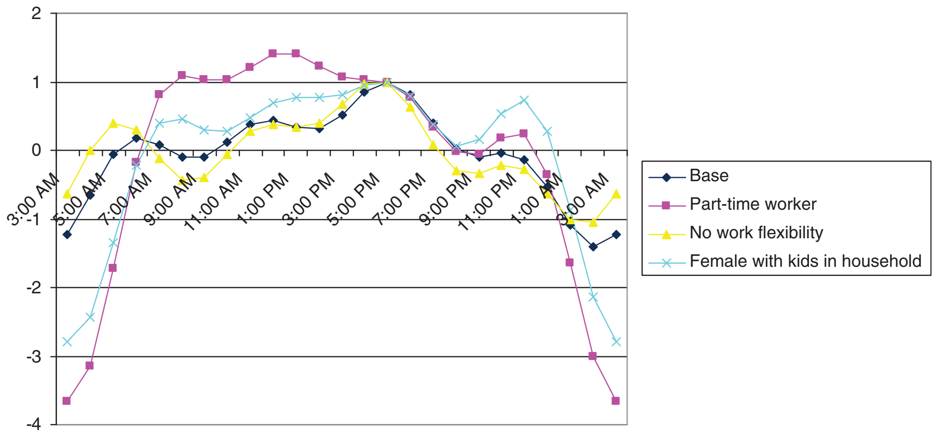

Figure 1. Departure time utility functions for the work tour model.

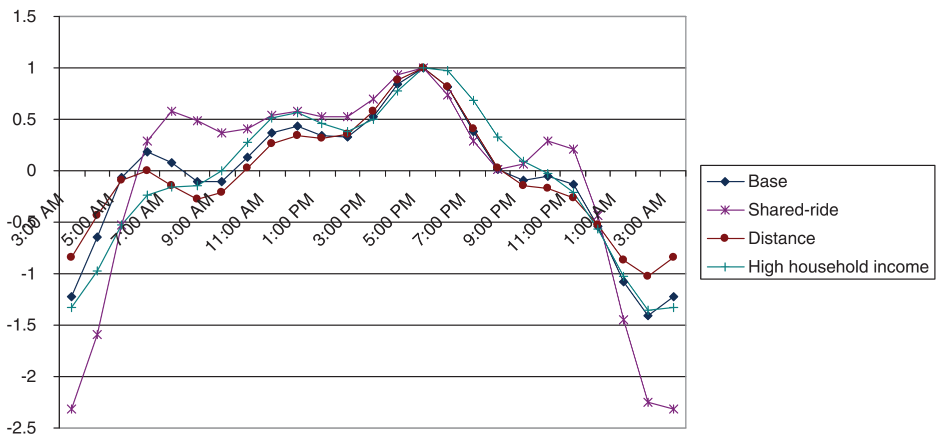

Figure 2. Departure time utility functions for the work tour model (cont.). For the distance plot, a distance of 10 miles is used.

The results shown in Figures 1 and 2 can be interpreted as follows. Compared to a full-time worker, the utility of departure from work for a part-time worker is larger between 6 AM and 5 PM and between 9 PM and 12 AM, which may be due to the nature of part-time jobs that may start in the morning and end in the early afternoon or occur during night shifts. The utility of departure is largest at 12 PM. The effect of work time flexibility on departure time utility is not very intuitive from the utility curve as time periods just before 5 PM seem to be more favourable to periods just after 5 PM for workers with no work time flexibility compared to those with work time flexibility. For a female with kids in the household, the utility of departure prior to 5 PM (between 7 AM and 5 PM) is larger than that of females without kids or of males, which is expected due to various responsibilities related to the kids (e.g. picking up kids from school, household chores), and is also larger for the period 9 PM to 1 AM.

For workers carpooling from work to home, the utility of departure before 5 PM is larger compared to workers driving alone from work to home and is smaller after 5 PM (except for the period 8 PM to 12 AM). Workers who carpool might not have the flexibility to stay late at work because of the constraints of the people carpooling with them. In the time periods surrounding 5 PM, the effect of the distance variable is to increase the utility of departure time periods prior to 5 PM; workers whose commutes are longer tend to depart from work earlier. Similarly, in the time periods surrounding 5 PM, workers with high household income tend to depart later relative to workers with low or medium household income, perhaps because the positions they hold require longer working hours.

6. Conclusion

This article has addressed three methodological issues related to time-of-travel modelling and proposed practical approaches for handling them in the context of operational travel demand model systems. The issues are unequal period lengths, schedule delay and the 24-hour cycle. We deal with the first issue by using size variables to account for time intervals of different lengths. Two methods are proposed for modelling schedule delay when desired times-of travel are unobserved. The first one is to use market segment-specific utility functions of time-of-travel and model arrival time choice for arrival-sensitive trips and departure time choice for departure-sensitive trips. The second one is to use a probability density function of the latent desired time-of-travel. The third issue is that the utility function of time-of-travel needs to be cyclical and can be modelled using a trigonometric function.

A case study was presented to demonstrate the time-of-travel modelling methodologies developed in this article. Tour-based time-of-travel models were estimated using the 2000 Bay Area travel survey, and selected model estimation results for work tours were presented. Thirty-five time periods, all consisting of 30-minute intervals except for two periods of longer duration, were used in the model. A trigonometric utility function was demonstrated. The approach developed here has also been used to estimate time-of-travel choice models for tours of other purposes both at the primary and secondary activity levels. The estimated models have then been incorporated in the San Francisco County Transportation Authority model, a microsimulation activity-based model, and used to test various scenarios such as highway and transit improvements and congestion pricing.

The tests showed that the time-of-travel distributions were reasonable and peak spreading was observed when congestion levels increased. Furthermore, the time-of-travel distributions predicted by the model for a baseline scenario compared favourably with the observed patterns.

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Note

1. In application, if short time intervals are used such that there is limited variation of the utility within an interval, then the exact choice of the 'mid-point' of the interval is not an issue.

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