MODELS OF FREEWAY LANE CHANGING AND GAP ACCEPTANCE BEHAVIOR

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ABSTRACT

Lane changing is an important component of microscopic traffic simulation models. In this paper a systematic approach for modeling lane changing behavior using a discrete choice framework is presented. Lane change is modeled as a sequence of three steps: decision to consider a lane change, choice of left or right lane, and search for an acceptable gap to execute the decision. Results from the estimation of the parameters of the gap acceptance model (using on-ramp merging data) show that in addition to the gap length, other important factors that affect drivers gap acceptance behavior are relative speed, distance remaining to the point at which lane change must be complete, and delay in completing merging.

INTRODUCTION

In response to the need for evaluating Intelligent Transportation Systems (ITS) strategies microscopic traffic simulation models are currently under development. ITS strategies include Advanced Traveler Information Systems (ATIS), Advanced Traffic Management Systems (ATMS), and Automated Highway Systems (AHS) -- see, for example, Ben-Akiva et al. (1994). An important element of simulation models, required for this evaluation, is the behavior of individual drivers in the presence of external stimuli. The main elements of this behavior are acceleration or car-following and lane changing decisions.
In this paper we focus on modeling lane changing behavior. Developing appropriate models that capture this behavior is important because the role of these models not only in microscopic traffic simulator but also in developing geometric standards, assessing capacity at weaving sections, and evaluating capacities of and delays at intersections and ramps.

Despite its importance, lane changing behavior has not been studied extensively. Though much focus has been placed on modeling the gap acceptance process, this represents only one aspect of lane changing behavior. Gipps (1986) presented a structure of lane changing decisions for urban roadways and examined situations in which drivers face conflicting goals. However, in this study driver behavior was modeled deterministically and the model parameters were not estimated formally. Yang and Koutsopoulos (1995) used a rule-based lane change model applicable for freeways only. A comprehensive list of scenarios were provided under which lane changing may be necessary or desirable, and priorities were explicitly modeled when drivers face conflicting goals. No formal estimation of parameters and validation of the models were conducted.

Gap acceptance, an important component of the lane changing process, has received more attention. Earlier efforts for modeling gap acceptance were based on the distribution of the critical gap (defined as the unobservable minimum gap a driver is willing to accept in order to merge) with no attempt to explain the underlying behavior -- see, for example, Herman and Weiss (1961), Miller (1972).

Daganzo (1981) used a probit model to estimate the parameters of a normal distribution of critical gaps at intersections and to capture the heterogeneity of driver behavior. This approach accounted for within driver variation, as well as across driver variation. However, the model had estimability problems; later research on dynamic modeling revealed that to identify the model one of the above-mentioned components of stochastic variation needs to be normalized -- see, for example, Heckman (1981). Mahmassani and Sheffi (1981) used a probit model to estimate the mean and the variance of critical gaps at unsignalised intersections, and concluded that the effect of number of gaps rejected on the critical gap was significant.

Kita (1993) formulated the problem of gap acceptance at merging points between on-ramps and freeways using a binary logit model with the gap length, remaining distance to the end of the acceleration lane, and relative velocity as explanatory variables. Cassidy et al. (1995) used a binary logit model to estimate the mean of a single valued critical gap function which can be used to empirically estimate capacity and delay. Different factors that affect gap acceptance behavior at intersections, such as the delay up to the occurrence of the gap under consideration and a first gap indicator, were also incorporated. In both cases, the potential serial correlation in a sequence of rejected gaps until one is accepted was not considered in the model formulation and estimation.

In this paper drivers' lane changing behavior on freeways is modeled in a systematic manner. The proposed model is a dynamic discrete choice model that captures the heterogeneity across the driving population as well as the above-mentioned serial correlation and is sensitive to design parameters and other factors that affect driver behavior.
METHODOLOGY

An observation of a lane change may be viewed as an outcome of the following sequence:

- decision to consider a lane change,
- choice of left or right lane, and
- accepting a gap in the desired lane.

The consequence of this structure, from a modeling and estimation point of view, is that, with the exception of a few special situations, the available data only capture the action of lane changing. The exact time at which a decision to consider a lane change takes place, or the state of looking for an acceptable gap, are not directly observable (they are latent). Freeway merging at on-ramps and merging or crossing a main street from a minor street (for example, at an unsignaledized intersection) are exceptions. In these situations a driver must seek an acceptable gap almost immediately upon arrival at the merge point or intersection, and the sequence of rejected and accepted gaps is observable.

In addition, lane changing may be mandatory or discretionary. Mandatory lane changing is performed when the current lane ceases to be an option (due to, for example, lane use regulations, incidents, and need to take exit ramps), and thus the driver must move to another lane. Discretionary lane changing is performed when a driver is not satisfied with the driving conditions in the current lane (due to, for example, average speed of the lane as compared to the driver’s desired speed, and existence of heavy vehicles).

The tree diagram in Figure 1 summarizes the proposed structure of the lane changing model.

![Figure 1 The Lane Changing Model Structure](image)
The ovals correspond to latent decisions while the rectangles correspond to events that are directly observable.

The top two levels capture the decision to consider a lane change. First, a driver who faces a situation that requires a mandatory lane change may respond to it immediately (MLC) or delay the response (MLC'). This binary decision is only relevant to drivers who face a mandatory lane change situation and is affected by explanatory variables such as the distance to the location at which the lane change must be complete, number of lanes to cross, and density of traffic. Short distances, many lanes to cross, and dense traffic make it more likely that a driver will respond to mandatory lane changing conditions immediately. If the driver decides not to respond temporarily to an existing mandatory lane changing situation or mandatory lane changing conditions do not apply (both of these situations are characterized by \( \overline{MLC} \)), the satisfaction with the current lane may be examined. That is, the driver may decide whether to consider a discretionary lane change (DLC) or not (\( \overline{DLC} \)). Factors that influence this decision may include speed differentials (defined as the difference between the speed of the vehicles ahead in various lanes and the desired speed of the driver considering a discretionary lane change), deceleration indicator (which captures the disutility when the lead vehicle in the lane under consideration decelerates), presence of heavy vehicle indicator (which captures the disutility associated with following a heavy vehicle), and ramp indicator (which captures the disutility for a lane adjacent to a ramp). Under the MLC and the DLC branches, when both the adjacent lanes are candidate lanes, the probability of considering each of the lanes is determined using the desired lane choice model. Important explanatory variables include speed differentials and indicators of deceleration, heavy vehicle, ramp, and need for a mandatory response (which captures the fact that when a mandatory lane change is required but the driver postpones the response, inappropriate lanes for mandatory lane changing conditions are not very desirable).

Finally, after a desired lane is selected, the driver seeks an acceptable gap. Explanatory variables, related to both the roadway environment and gap search characteristics, include relative speed, remaining distance to the point at which the lane change must be complete, first gap indicator (which captures the fact that the first gap that a driver encounters is not, for psychological reasons, as desirable as subsequent gaps), the number of gaps rejected, and the total delay experienced by the driver up to the occurrence of the gap under consideration. Even if an acceptable gap is found, the lane change may not occur immediately due to maneuvering time and reaction time. This is captured by a last stage of the model. If a lane change takes place immediately, the driver will be observed in a new lane; otherwise, the driver will continue in the current lane until a subsequent point in time when the lane change maneuver can be completed.

Based on the decision tree described above and assumptions regarding state dependence (dependence of current choice on previous experiences and decisions), the likelihood function corresponding to a set of observations can be formulated.

Assuming that observations of a driver's location and traffic conditions are available at discrete points in time (for example, every second), we denote the sequence of observations for a given driver as follows:

\[
(J_{1n}, J_{2n}, \ldots, J_{in}, \ldots, J_{T_{kn}})
\]
where,
\( J_n \) = index of the lane (Current, Left, or Right) driver \( n \) is observed at time \( t \); and,
\( T_n \) = number of time periods driver \( n \) is observed.

Generally, the \( T_n \) observations from individual \( n \) are likely to be correlated. To model this correlation, the random component of a random utility specification is assumed to have two elements: a random term attributable to a specific individual that does not change with \( t \) and a random term that varies across different time periods for a given individual, as well as across individuals (see Heckman, 1981). Therefore, the utility of a lane change at time \( t \) to driver \( n \) is written as follows:

\[
U_{in} = \gamma^T X_{in} + v_n + \epsilon_{in}
\]  

(1)

where,
\( U_{in} \) = utility for individual \( n \) at time \( t \);
\( X_{in} \) = vector of explanatory variables;
\( \gamma \) = vector of unknown parameters;
\( v_n \) = individual specific random term; and,
\( \epsilon_{in} \) = random term that varies across different time period for a given individual, as well as across individuals.

The following assumptions on \( v_n \) and \( \epsilon_{in} \) are made:

\[
\text{cov}(v_n, v_{n'}) = \begin{cases} 
\sigma_v^2 & \text{if } n = n' \\
0 & \text{if } n \neq n'
\end{cases}
\]

\[
\text{cov}(\epsilon_{in}, \epsilon_{i'n'}) = \begin{cases} 
\sigma_{\epsilon}^2 & \text{if } n = n', t = t' \\
0 & \text{otherwise}
\end{cases}
\]

\[
\text{cov}(v_n, \epsilon_{i'n'}) = 0 \forall t, n, n'
\]

where, \( \sigma_v^2 \) and \( \sigma_{\epsilon}^2 \) are the variances of \( v_n \) and \( \epsilon_{in} \), respectively. These assumptions imply:

\[
\text{cov}(U_{in}, U_{i'n'}) = \begin{cases} 
\sigma_v^2 + \sigma_{\epsilon}^2 & \text{if } t = t', n = n' \\
\sigma_v^2 & \text{if } t \neq t', n = n' \\
0 & \text{if } n \neq n', \forall t
\end{cases}
\]

Conditional on \( v_n \) and depending on the assumption on \( \epsilon_{in} \), different discrete choice models can be obtained, such as logit or probit.
Furthermore, the random terms associated with the same driver but at different nodes in the drivers' decision hierarchy are likely to be correlated. To capture this correlation without increasing the computational burden significantly, the driver specific random term is expressed as $\omega_{h} v_{n}$. The random variable $v_{n}$ is a generic random term, and the parameter $\omega_{h}$ (to be estimated) is associated with node $h$ (or level $h$) of the decision tree hierarchy.

With the above formulation of the random term, appropriate for panel data, the probability of observing a pattern for a given driver can be expressed, conditional on $v_{n}$ as follows:

\[
\text{Pr}(J_{1n}, J_{2n}, ..., J_{m}, ..., J_{T_{n}} | v_{n}) = \prod_{t=1}^{T_{n}} \text{Pr}(J_{tn} | v_{n}) \\
= \prod_{t=1}^{T_{n}} \text{Pr}_{t}(L | v_{n})^{\delta_{tn}^{L}} \text{Pr}_{t}(R | v_{n})^{\delta_{tn}^{R}} \text{Pr}_{t}(C | v_{n})^{\delta_{tn}^{C}}
\]

where,

\[
J_{tn} \in \{L, R, C\}
\]

$L$ = change to the left lane;

$R$ = change to the right lane;

$C$ = continue in the current lane;

$\delta_{tn}^{L} = \begin{cases} 1 & \text{if driver } n \text{ changes to the left lane at time } t \\ 0 & \text{otherwise} \end{cases}$

$\delta_{tn}^{R} = \begin{cases} 1 & \text{if driver } n \text{ changes to the right lane at time } t \\ 0 & \text{otherwise} \end{cases}$

$\delta_{tn}^{C} = \begin{cases} 1 & \text{if driver } n \text{ does not change lane at time } t \\ 0 & \text{otherwise} \end{cases}$

Finally, the likelihood function is given by

\[
L^{*} = \prod_{n=1}^{N} \int_{-\infty}^{\infty} \left( \prod_{t=1}^{T_{n}} \text{Pr}_{t}(L | v_{n})^{\delta_{tn}^{L}} \text{Pr}_{t}(R | v_{n})^{\delta_{tn}^{R}} \text{Pr}_{t}(C | v_{n})^{\delta_{tn}^{C}} \right) f(v_{n}) dv_{n}
\]

where, $f(v_{n})$ is the distribution of $v_{n}$ and $N$ is the sample size.

The probability $\text{Pr}(J_{tn} | v_{n})$ can be formulated from the decision tree of Figure 1. To formulate $\text{Pr}(J_{tn} | v_{n})$ for $J_{tn} = L$, for example, observe that there are two possible ways that a change to the left lane can be observed:

- A mandatory lane change is necessary and the driver is responding to it (MLC), the left lane is considered among the choices available, the gap in the left lane is acceptable, and the lane change takes place; or,
- A mandatory lane change is necessary but the driver is not responding to it (MLC) or mandatory lane change is not required, the driver is not satisfied with the current lane, the
left lane is considered among the choices available, the gap in the left lane is acceptable, and the lane change takes place.

Therefore, the probability of observing a \textit{change to the left lane} is

\[
\Pr(L | \mathcal{V}_n) = \Pr(\text{change lanes} | \text{gap acceptable, left lane chosen, MLC, } \mathcal{V}_n) \cdot \\
\Pr(\text{gap acceptable} | \text{left lane chosen, MLC, } \mathcal{V}_n) \cdot \\
\Pr(\text{MLC} | \mathcal{V}_n) + \\
\Pr(\text{change lanes} | \text{gap acceptable, left lane chosen, DLC, } \overline{\text{MLC}}, \mathcal{V}_n) \cdot \\
\Pr(\text{gap acceptable} | \text{left lane chosen, DLC, } \overline{\text{MLC}}, \mathcal{V}_n) \cdot \\
\Pr(DLC | \text{MLC}, \mathcal{V}_n) \cdot \Pr(\text{MLC} | \mathcal{V}_n) \tag{4}
\]

Similarly, \(\Pr(J_m | \mathcal{V}_n)\) for \(J_m = R\) or \(C\) can be formulated.

\textbf{CASE STUDY: MERGING FROM A FREEWAY ON-RAMP}

The application of the model presented in the earlier section is demonstrated for the case of merging from a freeway on-ramp. In this case, all drivers initiate the change to the adjacent mainline lane as soon as they cross the merge point between the on-ramp and the freeway lane (that is, everybody respond to the mandatory lane change requirement and the adjacent freeway lane is the only choice), and continue searching for acceptable gaps in the target lane. The above situation is a case of mandatory lane changing for which the right hand side of the tree presented in Figure 1 does not apply. The main elements of the decision process involve the acceptance of a gap and the actual lane change maneuver. Therefore, the lane changing model structure presented in Figure 1 reduces to the structure shown in Figure 2. There are two observable

![Figure 2. Lane changing model structure for the special case](image_url)
states: change to the left lane and continue in the current lane. A change to the left lane can be observed only when the gap is acceptable and lane change maneuver is completed. Otherwise, the driver will continue in the current lane.

Mathematical Formulation

Following Figure 2, the likelihood for this special case reduces to:

\[
L^* = \prod_{n=1}^{N} \int \left( \prod_{i=1}^{T_n} \Pr \left( L | v_n \right) \delta_{in} \Pr \left( Cl | v_n \right) \delta_{in} \right) f(v_n) dv_n
\]  

(5)

The probability that a driver will change to the left lane is given by

\[
\Pr_L(L | v_n) = \Pr_L(\text{change lanes} | \text{gap acceptable}, v_n) \cdot \Pr_L(\text{gap acceptable} | v_n)
\]  

(6)

and the probability that a driver will continue in the current lane is given by

\[
\Pr_C(Cl | v_n) = 1 - \Pr_L(L | v_n)
\]  

(7)

In formulating \( \Pr_L(L | v_n) \) therefore, the main elements are the formulation of the gap acceptance and the change lane models.

The proposed gap acceptance model addresses limitations of existing models, such as capturing heterogeneity and state dependence, and utilizing estimation methods appropriate for panel data.

In freeway operations, a gap is acceptable when both the lead and the lag gaps are acceptable. Figure 3 illustrates the definition of lead and lag gaps. The critical lead (lag) gap for a driver is defined as the unobservable minimum lead (lag) gap the driver is willing to accept in order to change lanes. It is assumed that the critical lead (lag) gap depends on traffic conditions and previous decisions made by the driver, and, therefore, is modeled as a random variable.

![Figure 3 The lead and the lag gap](image-url)
To capture the heterogeneity in the driver population, the critical gap $G_{cr,n}^g$ for driver $n$ at time $t$ is assumed to have the following form:

$$G_{cr,n}^g = \exp(\beta^g X_{in}^g + \nu_n^g + \epsilon_{in}^g)$$  \hspace{1cm} (8)

where,

- $g$ = index indicating whether the gap is a lead or a lag critical gap;
- $G_{cr,n}^g$ = value of the critical gap for driver $n$ at time $t$;
- $X_{in}^g$ = vector of explanatory variables;
- $\beta^g$ = vector of unknown parameters;
- $\nu_n^g$ = driver specific random term; and,
- $\epsilon_{in}^g$ = random term that varies across different components of a gap for a given individual, across different gaps for a given individual, as well as across individuals.

The assumptions on $\nu_n$ and $\epsilon_{in}$ made in formulating equation (1) apply here as well. In addition, for modeling the correlation between lead and lag critical gaps the following assumptions are made. The random terms $\nu_n^{lead}$ and $\nu_n^{lag}$ are expected to be correlated when they are associated with the same driver. This correlation is captured by setting $\nu_n^{lead} = \alpha_{lead} \nu_n$ and $\nu_n^{lag} = \alpha_{lag} \nu_n$, where, $\nu_n \sim N(0,1)$. These assumptions imply:

$$\text{cov}(\nu_n^{lead}, \nu_{n'}^{lag}) = \begin{cases} 
\alpha_{lead} \alpha_{lag} & \text{if } n = n' \\
0 & \text{if } n \neq n' 
\end{cases}$$

and $\nu_n^{lead}$ and $\nu_n^{lag}$ have different variances but are perfectly correlated. As for $\epsilon_{in}^g$,

$$\text{cov}(\epsilon_{in}^{lead}, \epsilon_{in'}^{lag}) = 0 \ \forall t, t', n, n'. \ \text{Finally, } \text{cov}(\nu_n^g, \epsilon_{in}^{g'}) = 0 \ \forall t, n, n', g, g'.$$

Hence, the lead and the lag components of the critical gap of an individual driver are correlated and critical gaps for the same driver over time are also correlated. The exponential form of the critical gap guarantees that the critical gap is non-negative.

Assuming $\epsilon_{in}^g \sim N(0, \sigma_{\epsilon_{in}}^2)$ (that is, the critical gap follows a lognormal distribution), the probability that the $g$-component of the $i^{th}$ gap observed by driver $n$, $G_{in}^g$, is acceptable is given by:
\[ \Pr(G_{in}^{k} \text{ acceptable } \mid v_n) = \Pr(G_{in}^{g} > G_{cr,in}^{g} \mid v_n) \\
= \Pr(G_{in}^{g} > \exp(\beta^{g} X_{in}^{g} + \alpha^{g} v_n + e_{in}^{g}) \mid v_n) \\
= \Pr(\ln(G_{in}^{g}) > \beta^{g} X_{in}^{g} + \alpha^{g} v_n + e_{in}^{g} \mid v_n) \\
= \Pr(e_{in}^{g} < \ln(G_{in}^{g}) - \beta^{g} X_{in}^{g} - \alpha^{g} v_n \mid v_n) \\
= \Phi \left( \frac{\ln(G_{in}^{g}) - \beta^{g} X_{in}^{g} - \alpha^{g} v_n}{\sigma_{e,g}} \right) \tag{9} \]

where, \( \Phi(*) \) denotes the cumulative distribution function of a standard normal random variable.

Therefore, the probability that the gap at time \( t \) is acceptable to driver \( n \) is

\[ \Pr(\text{gap acceptable to driver } n \mid v_n) \\
= \Pr(G_{in}^{lead} \text{ acceptable and } G_{in}^{lag} \text{ acceptable } \mid v_n) \\
= \Pr(G_{in}^{lead} > G_{cr,in}^{lead} \text{ and } G_{in}^{lag} > G_{cr,in}^{lag} \mid v_n) \\
= \Phi \left( \frac{\ln(G_{in}^{lead}) - \beta^{lead} X_{in}^{lead} - \alpha^{lead} v_n}{\sigma_{e,lead}} \right) \Phi \left( \frac{\ln(G_{in}^{lag}) - \beta^{lag} X_{in}^{lag} - \alpha^{lag} v_n}{\sigma_{e,lag}} \right) \tag{10} \]

Parameters to be estimated include \( \alpha^{lead}, \alpha^{lag}, \beta, \sigma_{e,lead}^{2}, \text{ and } \sigma_{e,lag}^{2}. \)

The change lanes model is assumed to be binary logit. The probability that lane change takes place at time \( t \), given the gap is acceptable, is as follows:

\[ \Pr(\text{change lanes } \mid \text{gap acceptable, } v_n) = \frac{1}{1 + e^{-\mu^{T} Z_n}} \tag{11} \]

where,

\[ Z_n = \text{vector of explanatory variables for driver } n \text{ at time } t; \]
\[ \mu^{T} = \text{vector of unknown parameters}. \]

Potential important explanatory variables include delay (time elapsed since the gap searching process began), remaining distance to the point at which lane change must be complete, and speed of the vehicle.

Data

For estimation of the parameters of the above model a data set collected by FHWA (Smith, 1985) was used. The data set consists of vehicle trajectories, recorded at discrete times, at a freeway site at I-95 NB near the Baltimore-Washington Parkway. Figure 4 shows a schematic diagram of the site. The site is 1606 feet long with a 695 feet weaving section, and was filmed at a rate of one snapshot per second. Vehicles traveling from the upstream end of the weaving section and merging with the mainline were considered for this special case of lane changing.
The data set includes observations on 286 drivers. Each observation corresponds to the trajectory of a vehicle traveling from the upstream end of the weaving section and merging with the mainline. The total number of gaps observed was 1447.

![Diagram of data collection site at I-95 NB near the Baltimore-Washington Parkway](image)

Figure 4  Data collection site at I-95 NB near the Baltimore-Washington Parkway

Estimation Results

The Maximum Likelihood estimates of the unknown parameters of the model are given in Table 1. The explanatory variables include the lag relative speed, remaining distance to the point at which lane change must be complete, first gap dummy, and delay in merging. Lag relative speed is the speed of the lag vehicle in the target lane less the speed of the subject vehicle. In this particular model, a piecewise linear approximation of the variable lag relative speed is used to capture the fact that the variable has a different impact on the critical gap length at different values (non linear relationship). First gap dummy equals one for the first second and zero otherwise. Delay in merging is the number of seconds elapsed since the gap searching process started.

The estimated lead and lag critical gaps (in feet) are:

\[
G_{cr,ln}^{lead} = \exp(2.72 - 0.055 v_n + \varepsilon_m^{lead}) 
\]

\[
G_{cr,ln}^{lag} = \exp(-9.32 + 1.17 \{ \min(V_{n,rel,lag}, 10)/10 \} + 1.174 \{ \max(0, V_{n,rel,lag} - 10)/10 \} + 1.88 \log_{10}(D_t) + 1.90 v_n + \varepsilon_m^{lat} 
\]

(12)

(13)

where,

\[
V_{n,rel,lag} = \text{relative speed in mph with respect to the lag vehicle at } t \\
= V_{n,lag,vehicle} - V_{n,subject} \\
D_t = \text{remaining distance to the point at which lane change must be complete in feet.} \]
The estimated model of changing lanes (using logit model), given that both the lead and the lag gaps are acceptable, is:

\[ Pr_t (\text{change lanes | gap acceptable, } v_n) = \frac{1}{1 + \exp(1.90 - 0.52(t - l))} \]  \hspace{1cm} (14)

### Table 1: Estimated parameters of the gap acceptance model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lead Critical Gap</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_{lead} )</td>
<td>-0.055</td>
<td>-0.19</td>
</tr>
<tr>
<td>( \sigma_{e,lead} )</td>
<td>1.61</td>
<td>4.54</td>
</tr>
<tr>
<td>constant</td>
<td>2.72</td>
<td>9.72</td>
</tr>
<tr>
<td><strong>Lag Critical Gap</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_{lag} )</td>
<td>1.90</td>
<td>4.15</td>
</tr>
<tr>
<td>( \sigma_{e,lag} )</td>
<td>1.31</td>
<td>1.79</td>
</tr>
<tr>
<td>constant</td>
<td>-9.32</td>
<td>-1.91</td>
</tr>
<tr>
<td>relative speed (10mph and less)/10</td>
<td>1.170</td>
<td>2.79</td>
</tr>
<tr>
<td>relative speed (above 10mph)/10</td>
<td>1.174</td>
<td>1.88</td>
</tr>
<tr>
<td>first gap dummy</td>
<td>1.57</td>
<td>1.36</td>
</tr>
<tr>
<td>log(e[remaining distance] (feet)</td>
<td>1.88</td>
<td>2.40</td>
</tr>
<tr>
<td>change lanes vs. no change lane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>-1.90</td>
<td>-6.12</td>
</tr>
<tr>
<td>delay (seconds)</td>
<td>0.52</td>
<td>3.91</td>
</tr>
<tr>
<td>Number of drivers = 286</td>
<td>Number of gaps observed = 1447</td>
<td></td>
</tr>
<tr>
<td>( L(0) = -706.7 )</td>
<td>( L(c) = -682.2 )</td>
<td></td>
</tr>
<tr>
<td>( L(\hat{\beta}) = -588.9 )</td>
<td>( \hat{\rho}^2 = 0.15 )</td>
<td></td>
</tr>
</tbody>
</table>

The lead critical gap was found to be insensitive to traffic conditions, whereas the lag critical gap was found to be a function of the relative speed (with respect to the lag vehicle in the target lane), remaining distance to the point at which lane change must be complete, and whether the gap is the first gap being considered or not. Variance of the lead critical gap was found to be significantly lower than that of the lag critical gap. The correlation between two different lead critical gaps, two different lag critical gaps, and between the lead and the lag critical gaps for a given individual were small.

A positive sign on the parameter of an explanatory variable implies that the critical gap increases as the value of the variable increases. The lag critical gap length increases as the lag relative speed increases. Higher sensitivity of critical gap at a higher lag relative speed is demonstrated by relatively higher estimated value of the parameter for the variable 'relative speed above 10mph' (compare to the variable 'relative speed equal to or below 10mph'). The initial hesitation of the drivers to merge into the mainline as soon as they appear at the upstream end of an acceleration lane is captured by the first gap dummy, which has the expected positive sign, implying a lower probability of acceptance of a gap observed at the first time period. As
the remaining distance to the merge point decreases, drivers become more aggressive and therefore are willing to accept smaller gaps -- the corresponding parameter has the desired positive sign. State dependence is captured by the variable delay. As drivers wait longer and longer in searching for gaps and trying to merge into the target lane they become impatient, and therefore, more aggressive -- this is reflected by the positive sign of the corresponding parameter of the variable delay in 'change lanes vs. no change lane'.

All the parameters, except the parameters of $\sigma_v$ for the lead critical gap and first gap dummy, have significant t-statistic at the 10% level of significance. The likelihood ratio test was used to test the null hypothesis that all the coefficients except the constants and the variance are zero. This statistic, $-2(L(c) - L(\hat{\beta}))$, is $\chi^2$ distributed with 5 degrees of freedom and is equal to 187. Hence, the null hypothesis can be rejected.

To assess how the proposed models replicate the actual gap acceptance process, the probability of acceptance of the gaps that were actually accepted and that the driver merged into were estimated for each driver. The probability that gap $t$ is acceptable to driver $n$ is:

$$
Pr(\text{gap}\ t \text{ acceptable to driver } n) = \int_0^\infty Pr(\text{gap}\ t \text{ acceptable to driver } n | V_n) f(V_n) dV_n
$$

$$
= \int_0^\infty \Phi\left( \frac{\ln(G_{m}^{\text{lead}}) - \beta_{\text{lead}} X_{m}^{\text{lead}} - \alpha_{\text{lead}} V_n}{\sigma_{e,\text{lead}}} \right) \Phi\left( \frac{\ln(G_{m}^{\text{lag}}) - \beta_{\text{lag}} X_{m}^{\text{lag}} - \alpha_{\text{lag}} V_n}{\sigma_{e,\text{lag}}} \right) f(V_n) dV_n \quad (15)
$$

where, $f(V_n)$ is the probability density function of a standard normal random variable. The average estimated probability was 0.61.

In addition, the median value of the lead and the lag critical gaps were calculated to see how they match the values found in the literature. The median value of the lead critical gap is unaffected by traffic conditions and is 15ft. The median value of the lag critical gap is sensitive to traffic conditions. Figure 5 shows how the median value of the lag critical gap varies according to the remaining distance for different values of the lag relative speed. For purposes of this discussion, let us assume that the lag vehicle speed is 50mph. For a 20mph lag relative speed, the median lag critical gap is 209ft (equivalent to 2.85sec. time headway) for a 700ft remaining distance. When the remaining distance is 300ft, the lag critical gap becomes 42ft (0.6sec. time headway). Similarly, for a given remaining distance, the median value of the lag critical gap progressively decreases as the lag relative speed decreases. When the lag relative speeds are negative (that is, the subject vehicle is traveling faster than the lag vehicle in the target lane) the median value of the lag critical gap, as expected, becomes very small.
CONCLUSIONS

The proposed integrated lane changing model captures the dynamics of the lane changing decision process. It models the underlying latent structure of the decision process and captures the stochasticity in driver behavior. The model system allows for a simultaneous estimation of the parameters of the lane change consideration, the lane choice, and the gap acceptance model. The gap acceptance model captures heterogeneity and state dependence, and recognizes the importance of both the lead and the lag gaps for freeway merging.
REFERENCES


